9 | The Product Formula

9.1 Theorem. *If* (X_1, x_1), (X_2, x_2) *are pointed spaces then*

$$\pi_1(X_1 \times X_2, (x_1, x_2)) \cong \pi_1(X_1, x_1) \times \pi_1(X_2, x_2)$$

9.3 Theorem. *If* $(X_i, x_i)_{i\in I}$ *is a family of pointed spaces then*

$$\pi_1\left(\prod_{i\in I} X_i,\ (x_i)_{i\in I}\right) \cong \prod_{i\in I} \pi_1(X_i, x_i)$$

9.4 Definition. Categorical product definition.

9.8 Definition. Let $F\colon \mathbf{C} \to \mathbf{C}'$ be a functor. Assume that F has the property that if an object d with morphisms $p_i\colon d \to c_i$ is the categorical product of a family $\{c_i\}_{i\in I}$ in $\mathbf{C}$ then the object $F(d)$ with morphisms $F(p_i)\colon F(d) \to F(c_i)$ is the categorical product of the family $\{F(c_i)\}_{i\in I}$ in $\mathbf{C}'$. In such situation we say the the functor F *preserves products*.

9.10 Theorem. *The fundamental group functor* $\pi_1\colon \mathbf{Top}_* \to \mathbf{Gr}$ *preserves products.*