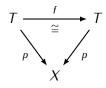
## 23 | Deck Transformations

**23.1 Definition.** Let  $p: T \to X$  be a covering. A *deck transformation* of p is an isomorphism of coverings



<b>23.2 Lemma.</b> Let $F: \mathbb{C} \to \mathbb{D}$ be a functor such that for any $c, c' \in \mathbb{C}$ the map the map $\mathrm{Mor}_{\mathbb{C}}(c, c') \to \mathrm{Mor}_{\mathbb{D}}(F(c), F(c'))$ given by $f \mapsto F(f)$ is a bijection. A morphism $f: c \to c'$ i in $\mathbb{C}$ is an isomorphism if and only if $F(f): F(c) \to F(c')$ is an isomorphism.
Proof. Exercise.
<b>23.3 Corollary.</b> Let $X$ be a connected and locally path connected space, $x_0 \in X$ , and let $p: T \to X$ be a path connected covering. The group of deck transformations $D(p)$ is isomorphic to the group of $\pi_1(X, x_0)$ -equivariant isomorphisms $p^{-1}(x_0) \to p^{-1}(x_0)$ .
Proof. Exercise.

**23.4 Definition**. Let G be a group, and let  $H \subseteq G$  be a subgroup. The *normalizer* of H in G is the subgroup  $N_G(H) \subseteq G$  defined by

$$N_G(H) = \{ g \in G \mid gHg^{-1} = H \}$$

**23.6 Proposition.** Let G be a group, and let S is a transitive G-set. For any  $s \in S$  there exists an isomorphism of groups

$$Iso_G(S) \cong N_G(G_s)/G_s$$

**23.7 Proposition**. Let X be a connected and locally path connected space, and let  $x_0 \in X$ . For a path connected covering  $p \colon T \to X$  and  $\tilde{x} \in p^{-1}(x_0)$  there exists an isomorphism of groups:

$$D(p) \cong N_{\pi_1(X,x_0)}(p_*(\pi_1(T,\tilde{x})))/p_*(\pi_1(T,\tilde{x}))$$