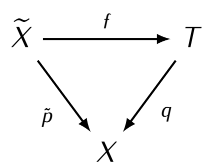
# 20 | From Subgroups to Coverings

$$\begin{pmatrix} \text{isomorphism classes} \\ \text{of path connected} \\ \text{coverings of } X \end{pmatrix} \xrightarrow{\Omega} \begin{pmatrix} \text{conjugacy classes} \\ \text{of subgroups} \\ \text{of } \pi_1(X, x_0) \end{pmatrix}$$

$$\begin{pmatrix} \text{isomorphism classes of} \\ \text{pointed path connected} \\ \text{coverings of } (X, x_0) \end{pmatrix} \xrightarrow{\Omega} \begin{pmatrix} \text{subgroups} \\ \text{of} \\ \pi_1(X, x_0) \end{pmatrix}$$

**20.1 Definition.** Let $X$ be a locally path connected space. A *universal covering* of $X$ is a covering $\tilde{p}\colon \widetilde{X} \to X$ such that $\widetilde{X}$ is a simply connected space.

**20.2 Proposition.** *Let $X$ be a locally path connected space and $\tilde{p}\colon \widetilde{X} \to X$ be a universal covering of $X$. For any covering $q\colon T \to X$ there exists a map of coverings:*



**20.3 Theorem.** *Let $X$ be a locally path connected space and let $x_0 \in X$. If there exists a universal covering $\tilde{p}\colon \widetilde{X} \to X$ then for each subgroup $H \subseteq \pi_1(X, x_0)$ there exists a covering $p_H\colon T_H \to X$ and $\tilde{x}_H \in p_H^{-1}(x_0)$ such that $p_{H*}(\pi_1(T_H, \tilde{x}_H)) = H$.*

**20.4 Definition.** A space $X$ is *semi-locally simply connected* if every point $x \in X$ has an open neighborhood $U \subseteq X$ such that the homomorphism $i_*\colon \pi_1(U, x) \to \pi_1(X, x)$ induced by the inclusion map $i\colon U \to X$ is the trivial homomorphism.

**20.7 Proposition.** *If $X$ is space such that there exists a universal covering $p\colon \widetilde{X} \to X$ then $X$ is semi-locally simply connected.*

*Proof.* Exercise. □

**20.8 Theorem.** *If $X$ is a space which is connected, locally path connected, and semi-locally simply connected then there exists a universal covering $p\colon \widetilde{X} \to X$.*