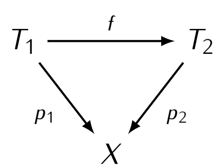
19 | Classification of Coverings

19.1 Definition. Let $p_1 \colon T_1 \to X$, $p_2 \colon T_2 \to X$ be coverings over the same base space X. A *map of coverings* is a continuous function $f \colon T_1 \to T_2$ such that the following diagram commutes:



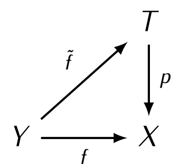
For a given space X by **Cov**(X) we will denote the category whose objects are all coverings over X and whose morphisms are maps of coverings.

19.2 Proposition. *Let $p_1 \colon T_1 \to X$ and $p_2 \colon T_2 \to X$ be coverings of X. A map of coverings $f \colon T_1 \to T_2$ is an isomorphism in* **Cov**(X) *if and only if f is a homeomorphism.*

Proof. Exercise. □

19.4 Theorem. *Let X be a locally path connected space, and for $i = 1, 2$ let $p_i\colon T_i \to X$ be a covering such that T_i is a path connected space. Let $x_0 \in X$ and let $\tilde{x}_i \in p_i^{-1}(x_0)$. The coverings p_1 and p_2 are isomorphic if and only if the subgroup $p_{1*}(\pi_1(T_1, \tilde{x}_1)) \subseteq \pi_1(X, x_0)$ is conjugate to the subgroup $p_{2*}(\pi_1(T_2, \tilde{x}_2))$.*

19.5 Theorem (Lifting Criterion). *Let $p\colon T \to X$ be a covering, let $x_0 \in X$ and let $\tilde{x}_0 \in p^{-1}(x_0)$. Assume that Y is a connected and locally path connected space and let $y_0 \in Y$. A map $f\colon (Y, y_0) \to (X, x_0)$ has a lift $\tilde{f}\colon (Y, y_0) \to (T, \tilde{x}_0)$ if and only if $f_*(\pi_1(Y, y_0)) \subseteq p_*(\pi_1(T, \tilde{x}_0))$.*



(∗)

19.8 Theorem. *Let (X, x_0) be a locally path connected space, and for $i = 1, 2$ let $p_i\colon (T_i, \tilde{x}_i) \to (X, x_0)$ be a pointed covering such that T_i is a path connected space. The coverings p_1 and p_2 are isomorphic in the category $\mathbf{Cov}_*(X, x_0)$ if and only if $p_{1*}(\pi_1(T_1, \tilde{x}_1)) = p_{2*}(\pi_1(T_2, \tilde{x}_2))$.*

Proof. Exercise. □