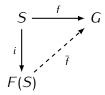
14 Presentations of Groups

14.1 Definition. Free group generated by a set S

14.3 Note. We will say that a group G is free if G is isomorphic to the group F(S) for some set S. Notice that by Theorem 13.13 the fundamental group of any 1-dimensional CW complex is free.

14.4 Theorem. Let S be a set and G be a group. For any map of sets $f: S \to G$ there exists a unique homomorphism of groups $\bar{f}: F(S) \to G$ such that the following diagram commutes:



14.5 Definition. Let S be a set, and let R be a subset of elements of the free group F(S). By $\langle S \mid R \rangle$ we denote the group given by

$$\langle S \mid R \rangle = F(S)/N$$

where N is the smallest normal subgroup of F(S) such that $R \subseteq N$. We say that elements of S are *generators* of $S \mid R$ and elements of $S \mid R$ are *relations* in $S \mid R$.

14.9 Definition. If G is a group and $G \cong \langle S \mid R \rangle$ for some set S and some $R \subseteq F(S)$ then we say that $\langle S \mid R \rangle$ is a *presentation* of G.

14.10 Definition. If a group G has a presentation $\langle S \mid R \rangle$ such that S is a finite set then we say that G is *finitely generated* and if it has a presentations such that both S and R are finite sets then we say that G is *finitely presented*.

14.11 Proposition. Every group has a presentation.

14.12 Note. 1) Every group has inifinitely many different presentations. For example

$$\mathbb{Z} \cong \langle a \rangle \cong \langle a,b \mid b \rangle \cong \langle a,b \mid ab^{-1} \rangle \cong \langle a,b \mid b^2,b^3 \rangle$$

2) In general if we know a presentation of a group it may be very difficult to say anything about the properties of the group (even if the group is trivial or not).